

A Mathematical Analysis of Washington Lottery Keno

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Introduction

This report provides a detailed mathematical analysis of the daily draw keno game offered through the Washington Lottery (<http://www.walottery.com/JackpotGames/DailyKeno.aspx>), hereafter called Washington Lottery Keno. Washington Lottery Keno is available at various retail establishments throughout the state of Washington (United States). It is a traditional 80-number keno game with a single pay table and no additional features. Players may wager fixed amounts between \$1 and \$20 per game and may play up to 25 consecutive games on a single ticket. Washington Lottery Keno has an aggregate payout limit of \$500,000 per game for players who hit all 10 spots on a 10-spot ticket.

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Game Overview

A blank Washington Lottery Keno game slip is shown in Figure 1. The player makes his or her selections on the front of the game slip. The back of the game slip explains how to complete the game slip and other pertinent information.

DAILY KENO
GET ON A ROLL

1 Select the spot you want to play
1 2 3 4 5 6 7 8 9 10

2 Select the dollar amount you want to play per drawing
1 2 3 4 5 10 20

3 Pick your numbers or...
Play **QUICK PICK**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

QUICK PICK **QP**

4 Select the number of consecutive drawings you want to play.
2 3 4 5 10 15 20 25

(Multiply number of consecutive drawings by dollars played per draw for total cost. Maximum cost per playslip is \$500.)

HOW TO PLAY DAILY KENO

1. Use only blue/black pen or pencil for marking playslip. Mark entry boxes like this ●
2. Select the spot you want to play. You may play the 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 spot.
3. Select the dollar amount you want to play: \$1, \$2, \$3, \$4, \$5, \$10, or \$20.
4. Select your own numbers based on the spot chosen, at least 1 number but not more than 10. For example, if you choose the 4 spot you must pick 4 numbers. Mark the "QP" box for quick pick if you want the computer to select your numbers.
5. To play your numbers for more than one drawing at a time, select the number of consecutive drawings you want to play. You may play up to 25 consecutive drawings. The ticket cost is equal to the dollar amount for the number of games played multiplied by the number of selected drawings. For example, if you played \$2 for 5 drawings, the ticket would cost \$10. Multiple draw tickets may be validated after each drawing. An exchange ticket will be issued for the remaining drawings(s).
6. Check your playslip carefully to make sure you have correctly marked the numbers you want before presenting to the retailer. This playslip is not a valid receipt.
7. Check your ticket(s) for accuracy before leaving the retailer's premises.

PRIZE PAYMENT

The prize levels vary with the dollar amount and the spot played. The overall odds of winning a prize are 1 in 16.6 or better.

DAILY KENO GAME RULES

Each ticket shall contain the spot and numbers selected, the drawing date(s), and the ticket cost. A ticket is the only valid proof of a play and the only valid receipt for claiming a prize. Check your ticket(s) for accuracy before leaving the retailer's premises. This playslip is not a valid receipt. Players agree to abide by the rules of Washington's Lottery. The holder of a winning ticket for a drawing may win only one prize per play but shall be entitled to the highest prize won by those numbers. A ticket is a bearer instrument. Neither Washington's Lottery nor its contractors shall be responsible for lost or stolen tickets.

Washington's Lottery has the right to publish information about prizes and prize winners. A prize claim form is a public record governed by Chapter 42.56 RCW. All Lottery games are played subject to Title 315 WAC and Chapter 67.70 RCW.

For Winning Numbers call: 1-800-545-7510
TDD/TTY users call: 360-586-0933

Washington's Lottery proceeds benefit scholarship programs for Washington students and other vital state programs.

walottery.com

Must be 18 or older to purchase.
Be a smart player, know your limit.
Problem Gambling Helpline: 1-800-547-6133.

WA 9004-03 B-1 FG 01/16

Figure 1 – Washington Lottery Keno Game Slip

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First, the player completes each section on the front of the game slip:

1. The number of spots to play, between 1 and 10.
2. The amount to wager per game: \$1, \$2, \$3, \$4, \$5, \$10, or \$20.
3. The numbers to play. There is also a “quick pick” option where the system randomly selects the numbers for the player.
4. The number of consecutive games to play: 2, 3, 4, 5, 10, 15, 20, or 25.

The player then gives the completed game slip and the total amount wagered to a clerk at the retail establishment. The clerk inserts the game slip into a terminal that scans the game slip and prints a corresponding bar-coded ticket like the one shown in Figure 2, which is given to the player. Some locations offer player-operated vending machines that dispense single game quick pick 10-spot tickets for \$1, \$2, \$5, or \$10.



Figure 2 – Washington Lottery Keno Ticket

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Pay Tables

Washington Lottery Keno uses a single pay table that offers the hit frequency of a traditional keno game, but considerably lower payouts. Figure 3 shows the pay table for a \$1 wager.

Play 1 Spot	
Hit	Award
1	\$2

Play 2 Spots	
Hit	Award
2	\$8

Play 3 Spots	
Hit	Award
2	\$2
3	\$16

Play 4 Spots	
Hit	Award
2	\$1
3	\$5
4	\$24

Play 5 Spots	
Hit	Award
3	\$2
4	\$17
5	\$200

Play 6 Spots	
Hit	Award
3	\$1
4	\$4
5	\$40
6	\$1,000

Play 7 Spots	
Hit	Award
3	\$1
4	\$2
5	\$10
6	\$100
7	\$2,500

Play 8 Spots	
Hit	Award
4	\$2
5	\$5
6	\$50
7	\$500
8	\$10,000

Play 9 Spots	
Hit	Award
4	\$1
5	\$5
6	\$10
7	\$100
8	\$2,500
9	\$25,000

Play 10 Spots	
Hit	Award
0	\$3
5	\$2
6	\$5
7	\$50
8	\$500
9	\$5,000
10	\$100,000

Figure 3 – Washington Lottery Keno Pay Table

If the player wagers more than \$1 per game, the above awards are multiplied by the wager amount. However, the award for hitting 10 out of 10 spots is an aggregate amount that is capped at \$500,000 per game. If the total payout for players who hit 10 out of 10 spots in the same game exceeds \$500,000, that payout is reduced to \$500,000 and is pro-rated between the winners according to the amount each player wagered in that game.

Calculating the Return to Player

The return to player (RTP) is the percentage of money taken in that is paid back to the player for all possible outcomes of a game. For example, a game with an RTP of 94% means that over the long haul, one can expect that 94% of the money taken in is returned to the player, and 6% is retained. It does *not* mean that if a single player makes wagers totaling \$100 that he can expect to receive \$94 in payouts.

To calculate the RTP for a keno game, we must first calculate the probability of achieving each possible outcome for the number of spots played. For example, if we are calculating the RTP for a 4-spot keno ticket, we first calculate the probability of hitting exactly 0, 1, 2, 3, and 4 spots. This is easiest done using a branch of mathematics called *combinatorics*, which includes a way to calculate the number of ways to choose k objects from a group of n objects. This number, read aloud as “ n choose k ”, is often represented by the notation:

$$C \binom{n}{k}$$

and is calculated by the formula:

$$C \binom{n}{k} = \frac{n \times (n - 1) \times (n - 2) \times \dots \times (n - k + 1)}{k \times (k - 1) \times (k - 2) \times \dots \times 1}$$

So if N is the number of spots played and H is the number of spots hit, the probability of hitting exactly H of N spots is given by this formula:

$$\frac{C \binom{20}{H} \times C \binom{60}{N - H}}{C \binom{80}{N}}$$

Returning to our 4-spot example, let’s first calculate the probability of hitting exactly 3 out of 4 spots. The above formula states that this probability is the number of ways to choose 3 numbers from the 20 called numbers, multiplied by the number of ways to choose 1 number (4 minus 3) from the remaining 60 numbers, divided by the total number of ways to choose 4 numbers from the domain of 80 numbers. This yields a probability of 0.0432479, or a frequency of 4.32479%.

Next, we need to know the RTP contribution for hitting 3 out of 4 spots. This is simply the frequency of that outcome multiplied by the associated award. In the Washington Lottery Keno pay table, hitting 3 out of 4 has an award of 5 for 1. So the RTP contribution for hitting 3 out of 4 spots is 4.32479% multiplied by 5, or 21.62%.

To calculate the overall RTP for a 4-spot keno ticket, we calculate the RTP contribution for each outcome – that is, hitting 0, 1, 2, 3, and 4 spots – and sum those individual contributions to get the overall RTP. Figure 4 shows the individual RTP contributions and the overall RTP for a 4-spot ticket in Washington Lottery Keno.

Play 4 Spots			
Hits	Frequency	Award	RTP
0	30.83214%	0	0.00%
1	43.27318%	0	0.00%
2	21.26355%	1	21.26%
3	4.32479%	5	21.62%
4	0.30634%	24	7.35%
Total	100.00%		50.24%

Figure 4 – RTP Calculations for Playing 4 Spots

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So Washington Lottery Keno has an RTP of 50.24% for the 4-spot ticket – not exactly favorable.

It should be mentioned that the frequency of hitting a given number of spots is completely independent of the pay table, and that the frequencies for all cases must total 100%. Because we can't change the frequency of each keno outcome, we must change the award for each outcome (that is, the pay table) if we want to change the RTP.

Calculating the Hit Frequency

The hit frequency is the percentage of games where the player wins something, even if the amount won is less than the amount wagered. Analogous to RTP, a game with a hit frequency of 25% means that over the long haul, one can expect that 25% of the games played will result in a payout. It does not mean that a single player who plays 100 games will receive a payout in 25 of those games.

To calculate the hit frequency for a keno game, we can use the frequencies calculated when determining the RTP. We simply sum the frequencies where the associated award is greater than zero. So referring back to Figure 4 for our 4-spot keno, the player wins something when hitting 2, 3, or 4 out of 4 spots, and the frequencies for those outcomes are 21.26355%, 4.32479%, and 0.30634% respectively. Summing those three values and rounding the result to two decimal places gives a hit frequency of 25.89% for a 4-spot keno ticket.

Hit frequency can also be expressed as the odds of receiving a payout in a game, which is simply the reciprocal of the hit frequency percentage. So for the 4-spot keno ticket, this is $1 / 25.89$, or 3.86. We can thus say that the chance of winning something is 1 in 3.86, or 1:3.86, which matches the published odds for a 4-spot ticket shown on the Washington Lottery website.

Hit Frequency and Return to Player Summary

Figure 5 shows the hit frequency and RTP values for playing 1 through 10 spots in Washington Lottery Keno. These values were calculated as described for the 4-spot ticket in the previous two sections.

Spots Played	Hit Freq	Once Per	RTP
1	25.00%	4.00	50.00%
2	6.01%	16.63	48.10%
3	15.26%	6.55	49.95%
4	25.89%	3.86	50.24%
5	9.67%	10.34	50.24%
6	16.16%	6.19	49.68%
7	23.66%	4.23	50.00%
8	10.23%	9.77	49.65%
9	15.31%	6.53	49.31%
10	11.05%	9.05	48.77%
Average	15.82%	6.32	49.59%

Figure 5 – Hit Frequency and RTP Summary

As shown in Figure 5, the RTP values range from a low of 48.10% for a 2-spot ticket to a high of 50.24% for a 4-spot or 5-spot ticket, with an average RTP of 49.59% for 1 through 10 spots. Figure 5 further reveals that the hit frequencies range from a low of 6.01% for a 2-spot ticket to a high of 25.89% for a 4-spot ticket, with an average hit frequency of 15.82% for 1 through 10 spots.

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The Washington Lottery does not publish RTP values, but it does provide hit frequency data for playing 1 through 10 spots on the Washington Lottery website. The published hit frequencies are rounded to three significant digits and are expressed in the “once per” format. In addition, verbiage on the dispensed keno ticket and on the back of the game slip states that the overall odds of winning a prize are 1 in 16.6 or better. Once per 16.6 is the hit frequency for a 2-spot ticket; it has the lowest hit frequency. The published hit frequencies for playing 1 through 10 spots exactly match those in Figure 5.

Payout Limit and its Effect on the RTP

Earlier we mentioned that Washington Lottery Keno has an aggregate payout limit of \$500,000 per game for players who hit all 10 spots on a 10-spot ticket. Since hitting 10 out of 10 pays \$100,000 for a \$1 wager, and hence \$500,000 for a \$5 wager, any wager of more than \$5 per game is subject to this limit. Lesser wagers might also be subject to this limit if multiple players hit 10 out of 10 in the same game, but we will ignore that rare case in this discussion.

The only wager amounts that Washington Lottery Keno allows above \$5 per game are \$10 and \$20 per game. For \$10 and \$20 wagers, the award for hitting 10 out of 10 is \$500,000. So in such cases, the award becomes \$50,000 per \$1 bet for a \$10 wager, and \$25,000 per \$1 bet for a \$20 wager. Since the award for hitting 10 out of 10 is normally \$100,000 for a \$1 wager, the 10-spot RTP is effectively reduced when wagering \$10 or \$20 per game. Figure 6 summarizes the effective RTP for a 10-spot ticket for all allowed wagers.

Wager	Award Per \$1 Bet	RTP Contribution		Effective RTP
		Hit 0 to 9 Spots	Hit 10 Spots	
\$1 to \$5	100,000	47.65%	1.12%	48.77%
\$10	50,000	47.65%	0.56%	48.21%
\$20	25,000	47.65%	0.28%	47.93%

Figure 6 – Effective RTP for a 10-Spot Ticket

The RTP contributions shown in Figure 6 for hitting 0 to 9 spots on a 10-spot ticket are always 47.65% because the associated awards are not constrained to any payout limit. But for hitting 10 out of 10 spots, the RTP contribution is the frequency of hitting 10 out of 10 (0.00001122%) multiplied by the award amount for a \$1 bet. Because the effective award amounts are reduced for a \$10 or \$20 wager, so is the associated RTP contribution.

Figure 5 showed the Washington Lottery Keno average RTP is 49.59% for playing 1 through 10 spots. That value is correct for wagers not subject to the 10 out of 10 payout limit (that is, wagers of \$5 or less per game). When we consider this limit and use the effective 10-spot RTP values shown in Figure 6, the average RTP becomes 49.54% for a \$10 wager and 49.51% for a \$20 wager.

Player Simulation Metrics

To quantify the Washington Lottery Keno player experience, computer simulations of 100,000 players were performed when playing 1 through 10 spots. In the simulations, each player wagered \$1 per game to avoid the effects of the payout limit when playing 10 spots. Each player was given a \$20 starting bankroll and continued playing until they played 600 games or until their bankroll was exhausted, whichever occurred first. Figure 7 shows the average number of games played in each simulation. As this data shows, the average number of games roughly ranges from 32 to 38 regardless of the number of spots played.

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Spots	Avg Games
1	38.04
2	36.60
3	38.01
4	38.18
5	37.54
6	32.24
7	34.33
8	33.00
9	31.93
10	32.77

Figure 7 – Average Games Played Simulation Metrics

Since each of the 100,000 simulated players had a \$20 starting bankroll and wagered \$1 per game, each player was guaranteed to play at least 20 games. Figure 8 shows the percentage of the players remaining at six different levels – from the 20-game minimum to the 600-game maximum – for each simulation. For example, when playing 1 spot, 99.57% of the players were able to play more than 20 games (or put another way, 0.43% of such players won nothing and exhausted their bankroll in 20 games).

Spots Played	Simulation Hit Freq	Games Played					
		> 20	> 30	> 50	> 100	> 200	600
1	25.03%	99.57%	76.28%	12.87%	0.02%	0.00%	0.00%
2	6.01%	69.27%	46.48%	21.41%	2.00%	0.06%	0.00%
3	15.27%	95.70%	43.99%	19.17%	3.03%	0.15%	0.00%
4	25.94%	98.31%	54.79%	15.72%	2.15%	0.09%	0.00%
5	9.70%	85.56%	27.55%	10.25%	2.85%	1.98%	0.35%
6	16.15%	87.86%	19.51%	7.87%	1.86%	0.51%	0.39%
7	23.66%	97.45%	33.06%	6.02%	2.29%	0.85%	0.10%
8	10.23%	87.15%	20.95%	6.39%	2.53%	0.71%	0.51%
9	15.31%	86.50%	31.84%	5.21%	1.81%	0.58%	0.11%
10	11.04%	88.93%	27.76%	5.12%	2.40%	0.53%	0.42%
Average	15.83%	89.63%	38.22%	11.00%	2.09%	0.55%	0.19%

Figure 8 – Time on Device Simulation Metrics

As one would expect, there is a correlation between the hit frequency and the time on device. Looking at some specifics, the 2-spot and 5-spot tickets have the lowest hit frequencies, while the 1-spot and 4-spot ticket have the highest. The player simulation results show the 2-spot and 5-spot tickets have the lowest percentage of players to play more than 20 games (69.27% and 85.56%), while the 1-spot and 4-spot tickets have the highest (99.57% and 98.31%).

The simulation results in Figure 8 also show that for 1-spot through 4-spot tickets, all players exhausted their bankrolls before reaching 600 games. This is because the maximum award is relatively low when playing 1 through 4 spots, and even hitting the maximum award a few times isn't nearly enough to overcome the low RTP.

Game Strategy

The best strategy for Washington Lottery Keno is not to play. But kidding aside, here are some basic guidelines to help you enjoy the game.

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- The RTP values for playing 1 through 10 spots are approximately the same, so there really is no mathematical reason to favor playing a specific number of spots.
- Don't wager more than \$5 per game if you're playing a 10-spot ticket (to avoid the \$500,000 payout limit).
- Head across the river and play the keno game offered through the Oregon Lottery, which has a still low but more favorable 68% RTP.

Conclusions

The Washington Lottery does not publish RTP values for Washington Lottery Keno, but our calculations showed an average RTP of 49.59%. This RTP is well below that of typical video keno games, which commonly have RTP values between 85% and 95%. We can attribute some of this difference to the overhead of running a daily draw lottery game and the fact that retailers receive a 5% commission on lottery sales. Still, even with these expenses, 49.59% is a very low RTP for any type of game.

About the Author

Ted Gruber is president and co-founder of Ted Gruber Software, Inc., (TGS) a Nevada corporation specializing in the design and development of mathematical models for the gaming industry. TGS specializes in math for slot machines, bingo, keno, video poker, scratch cards, pull tabs, table games, side bets, skill games, gaming promotions, new games – you name it.

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